

TPMS-Optimized Cement Geometries for Carbon-Efficient Structural Performance

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Submitted: 31 July 2026 Accepted: 09 August 2026 Publication date: 18 September 2026

DOI: 10.70671/dq9am588

Abstract: Cement production generates 8% of global anthropogenic CO₂ emissions, and concrete's low tensile strength (1/10th of its compressive strength) leaves structures vulnerable to seismic failure. This study shows that topology-optimized Triply Periodic Minimal Surface (TPMS) geometries can simultaneously reduce cement consumption and improve structural resilience without altering material chemistry. We generated and computationally evaluated 8,750 unique TPMS configurations, two orders of magnitude more than in prior studies, across the full structural porosity range (0–70%) by varying seven topological families, wall thickness, radial porosity gradients biomimicked from trabecular bone, and span. All structures were simulated using Mazars continuum damage mechanics; top performers then underwent Newmark- β seismic simulations, with physical validation via a calibrated shaker table. Retaining 93% of the solid's compressive strength, Lamella was the optimal topology, reducing mass by 32% and achieving 139.9% of the control's strength-to-weight ratio, along with an 180x improvement in the tensile strength-to-weight ratio. All seven topologies achieved an 85–98% reduction in seismic response amplification and a 50–88% reduction in peak stress, shifting the failure mode from brittle crushing to distributed energy absorption. Dual machine learning frameworks enabled predictive optimization ($R^2 = 0.987\text{--}0.999$) and the generative synthesis of hybrid topologies that achieved 104% of the best classical compressive performance. These results offer a component-level CO₂ reduction pathway requiring no supply-chain or workforce changes while democratizing seismic resilience at the material level for earthquake-prone, low-income communities.

Author keywords: Engineering; structural engineering; triply periodic minimal surfaces; topology optimization; sustainable concrete; seismic resilience

Introduction

The building and construction sector contributes significantly to CO₂ emissions. Because of this, demand for sustainable and efficient building practices has increased over the past 10 years.^{1,2} The construction sector alone is responsible for approximately 40% of solid waste and 37–38% of greenhouse gas emissions.^{1–3} Cement alone is responsible for around 8% of anthropogenic CO₂ emissions.^{4,5} One of the primary reasons for this is the sheer amount of cement used. Within the next 30 years, it is estimated that there will be more cement than biomass on the surface of the Earth.⁶

The primary type of cement, Portland cement, is difficult to replace because of its mechanical performance and abundance. Recent research has focused on alternative binders, such as biocement formulations, geopolymers, and alkali-activated materials. However, these alternatives have not achieved the widespread adoption necessary to replace

conventional cement at scale.^{7–10} The difficulty with these materials lies in replicating the unique properties that have made Portland cement the backbone of global construction infrastructure (Bye). Ultimately, reducing cement's CO₂ emissions by making a “greener” mix remains, as of now, unrealistic: chemical additives modify the pore structure and ion-leaching behavior, which may compromise long-term durability for short-term strength gains, while even slight nano-additions can raise water demand and increase yield stress.^{11–13}

Another strategy to decrease the construction sector's CO₂ emissions is to reduce the amount of material used.^{6,14} Despite growing research in topology optimization (TO) for building envelopes, several gaps remain that limit its application to sustainable construction. Most TO studies are space-specific, making them less generalizable for broader applications or prefabricated building components.^{15,16}

There is currently no material-level, homogenization-based topology-optimization (TO) framework for cement.¹⁷ TO has already been applied to metal and polymer additive-manufacturing parts, where homogenized TPMS models allow for more efficient structural design, and metallic TPMS lattice studies demonstrate substantial performance gains.¹⁸ A homogeneous TPMS architecture could lower the

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amount of cement required while preserving the intrinsic strength of Portland cement. This offers a scalable, low-CO₂ solution that current chemical-additive or space-specific TO approaches cannot provide. Developing a universal, topology-optimized material system, rather than relying on space-specific geometries, would reduce design and computational costs.

The objective of this study is to determine the optimal TPMS-based internal geometries for cementitious materials that maximize strength-to-weight ratio and seismic resilience, as well as to mimic trabecular bone's hierarchical load distribution and adaptive porosity gradients through TPMS structures.

Success would allow for material-level seismic resilience without expensive structural systems (moment frames, dampers), thereby democratizing earthquake protection for low-income communities while reducing construction-sector CO₂ emissions through material savings.

Topology optimization

Although topology optimization is widely used in engineering to design components with optimal strength and stiffness, it has seen little application in the design of cement and cementitious structures.¹⁶ The use of less material can decrease emissions with no loss of integrity in a structure. The improvements in 3D printing for concrete construction (3DCP) have made it much more viable to mimic bio-inspired structures, including lattices.^{19,20} 3DCP uses the additive layering of liquid concrete to build structures, allowing for new freedoms in the design of cement geometries.

Concrete failure mechanisms: tensile vs. compressive failure modes

Concrete exhibits distinct responses under tensile and compressive loading. Under tensile loading, brittle fracture occurs almost instantaneously once the tensile capacity is reached, followed by a near-complete loss of load-carrying capacity. This response is typically characterized by localized macro-scale cracking. Because cracking tends to occur abruptly and in a relatively discrete manner, the tensile stress-strain response remains approximately linear until fracture.²¹

Under compressive loading, concrete exhibits a more distributed damage process. This response can be described as a three-stage progression: (1) microcrack initiation, (2) progressive microcrack accumulation, and (3) sudden crushing. Damage is often concentrated at interfacial transition zones (ITZs) between aggregates and cement paste, where material heterogeneities promote stress concentrations.^{22,23}

Material-level sustainability: recycled aggregates and limitations

Material sustainability approaches in concrete, such as fine recycled concrete aggregates, have been explored in the literature but are inconsistent and face standardization gaps, making them hard to implement.²⁴ Many of these solutions

also face increased water absorption that limits sustainable scalability, creating an argument for geometry-driven optimization.²⁴ The use of fine recycled concrete aggregates (fRCA) in cement leads to inconsistent performance because of variability in the physical and chemical properties of fRCAs. Adhered mortar on fRCA particles also increases their porosity, leading to higher water absorption rates, necessitating the creation of an entirely different cement mix with more water, and increasing freshwater use.¹

Triply periodic minimal surfaces (TPMS)

Triply periodic minimal surface (TPMS) lattices are interconnected and non-intersecting surfaces with three-dimensional periodicity.²⁵ They also have higher mechanical energy absorption than traditional lattice structures at similar densities.²⁵ TPMS have been observed in nature as biological membranes, block copolymers, and equipotential surfaces in crystals, among other systems. Because of their properties, they have been used to model bone substitutes. They have also been of interest in architecture, design, and art.²⁶

Biological inspiration: trabecular bone architecture

This project creates Triply Periodic Minimal Surface (TPMS) lattices inspired by the hierarchically porous architecture of trabecular bone to mimic the phenomenon of strength retention in very brittle materials. Trabecular bone is made of hard, 3D, stiff yet ductile struts that allow the spaces between them to be filled with bone marrow. In this project, we attempt to replicate this phenomenon using TPMS by varying the configuration of these surfaces and modifying the porosity, wall thickness, and gradients to examine different combinations and achieve the best stress distribution in brittle cement-based materials.

Seismic performance through mass-stiffness decoupling

Beyond CO₂ reduction, lowering reliance on cement can produce advantageous structures. Although cement and concrete are ubiquitous, they are not as strong as required to withstand the natural disasters and climate change challenges we are all facing. While concrete can withstand great compressive force, its brittle nature, resulting from its lack of tensile strength, means that it can fail easily. Natural disasters exert extreme and unpredictable forces on the environment, which cause the chronic deterioration and eventual failure of concrete to remain intact, and will cause it to spall and crack under tension.^{27–30}

Earthquake concerns directly correlate to socioeconomic status. In poorer regions, the dominant forms of housing consist of unreinforced masonry and non-ductile concrete structures that completely lack consideration of seismic safety.^{31,32} These poorer neighborhoods are often surrounded by commercial districts and higher-income neighborhoods that are built under higher safety standards and incorporate moment-resisting frames and shear walls,

along with foundation systems that control the liquefaction of soil.³³ These commercial districts and higher-income neighborhoods are likely to experience far less seismic structural failure than poorer neighborhoods with non-ductile concrete structures. Incorporating built-in seismic resistance will help distribute the energy of future seismic activity.

When solid cement is replaced by engineered TPMS lattices (gyroid, Schwarz-P), the mass of the element decreases while axial stiffness is largely preserved because load-bearing struts align with principal stress paths. For a simplified single-degree-of-freedom system, the natural frequency is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

This indicates that the resonant frequency of a structure increases as its mass decreases while stiffness is held roughly constant. This method helps avoid the 0.5–3 Hz band, as most earthquake-induced spectral accelerations are concentrated in this frequency band. This also helps minimize spectral acceleration demand (Sa), thereby reducing the inertial force of the structure ($F = ma$), since base shear demand ($V = CW$) is also reduced. This is caused by the decrease in the seismic coefficient C and a decrease in the structural weight W , which is caused by a decrease in the mass of the structure.

Also, a decrease in mass reduces the kinetic energy ($E_k = \frac{1}{2}mv^2$) that the structure possesses. Deformations caused by shear wall yielding and the formation of plastic hinges become less probable, as does brittle cracking in concrete that is unable to undergo yielding. TPMS geometries promote a strong, interconnected structure that retains the shear stiffness of the lightweight, low-density TPMS structure while maintaining a good level of stiffness compared to solid concrete. This minimizes structural drift while also reducing the effects of seismic activity on the structure. However, the beneficial effects of the frequency shift in a seismically resilient structure are within the spectral range of activities caused by an earthquake. Improved seismic resilience is a result of the cement-reduced TPMS lattices, in which mass is decoupled from stiffness. This decreases inertia without significantly affecting load resistance and energy dissipation.

Research gaps and critical limitations

Currently, topology optimization has three major gaps: (1) Most studies are site-specific. These studies optimize topology for site-specific constructions, resulting in site-specific constructions. They fail to produce homogeneous, universal constructions.^{15,16} (2) The reviewed articles concerning TPMS research operate in the 7–30% density range. The low- and high-density domains are significant in the full context of the strength-to-density relationship of the geometry and therefore need to be examined. (3) The reviewed studies did not consider the effects of different TPMS topologies on cement (gyroid, Schwarz-P, Schwarz-D, diamond, neovius, and lidinoid) under the same testing conditions and with the same geometric set.

Description/Methodology/Proposed Research Approach

Parametric data generation and simulation

Through the systematic iteration of four key parameters hypothesized to influence structural performance, this study generated a data set of approximately 9,000 unique TPMS structures. Each structure underwent computational simulations evaluating compressive strength, tensile strength, and seismic resistance, thereby composing the data set used to train an AI model.

The investigated parameters include:

- 1) TPMS structure type (gyroid, Schwarz primitive, Schwarz gyroid, lidinoid, diamond, neovius, and lamella)
- 2) Span of each cell
- 3) Radial porosity gradient (a threshold-defined transition between a dense core and a porous shell, or the inverse configuration)
- 4) Wall thickness (strut diameter at constant porosity).

STL generation

The equations of the TPMS structures used in this study are as follows (Fig. 1):

Gyroid:

$$\cos(x) \sin(y) + \cos(y) \sin(z) + \cos(z) \sin(x)$$

Schwarz Primitive:

$$-(\cos(x) + \cos(y) + \cos(z))$$

Schwarz Gyroid:

$$\sin(x) \cos(y) + \sin(z) \cos(x) + \sin(y) \cos(z)$$

Lidinoid:

$$\frac{1}{2} \sum_{cyc} \sin(2x) \cos(y) \sin(z) - \frac{1}{2} \sum_{cyc} \cos(2x) \cos(2y)$$

Diamond:

$$\sin(x) \sin(y) \sin(z) + \sum_{cyc} \sin(x) \cos(y) \cos(z)$$

Neovius:

$$3(\cos(x) + \cos(y) + \cos(z)) + 4(\cos(x) \cos(y) \cos(z))$$

Lamella:

$$\frac{1}{2} \sum_{cyc} \sin(x) \cos\left(\frac{y}{2}\right) \sin\left(\frac{z}{2}\right) - \cos(z) \cos(x)$$

This project uses an implicit surface approach to generate STL files: a mathematical TPMS function is evaluated on a 3D grid, thresholded to create a binary volume, and a mesh is then extracted. The script defines a voxel grid

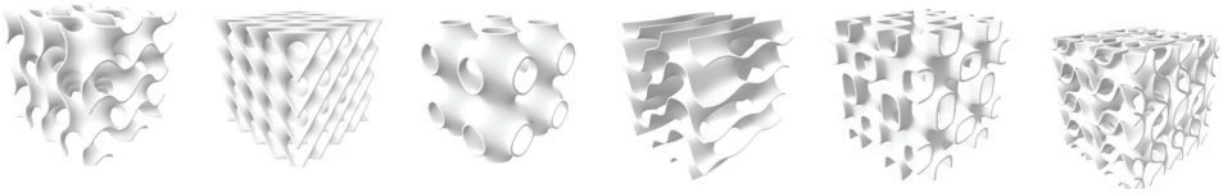


Figure 1. TPMS topology families used in this study. From left to right: Gyroid, Diamond, Schwarz Primitive, Lamella, Neovius, and Lidinoid. All structures are shown unscaled and at uniform wall thickness

covering the domain of the region, with two parameters: (a) subdivisions—voxels per cell, defining the resolution (this project implemented 150 subdivisions per structure), and (b) the physical size (mm) per cell (this project defined the unit cell size as 10 mm and developed a $3 \times 3 \times 3$ tiling of cells). The grid size equals ($\text{numx} \times \text{nsteps}$, $\text{numy} \times \text{nsteps}$, $\text{numz} \times \text{nsteps}$). Given a TPMS equation from above, the library computes the TPMS function at every point and normalizes it to $[-1, 1]$. This normalization is then used to convert the continuous field into a binary volume (material/void). Finally, the marching cubes algorithm extracts a triangular mesh, creating a smooth surface from discrete voxels, which is exported as an STL file.

Porosity gradient

Rather than assigning a uniform porosity throughout the geometries, porosity was varied as a function of radial distance from the mesh center. This was inspired by prior work, notably Zhang et al., on graded gyroid scaffolds for bone tissue engineering, where density gradients were developed, mimicking trabecular bone growth.³⁴

Two radial gradient configurations were examined: (1) forward radial gradient, with a dense core and porous exterior; and (2) reverse radial gradient, with a porous core and dense exterior. Using the same continuous field and STL developed in the previous step, a script was developed to impose a radial porosity gradient. The radial distance of each voxel from the geometric center of the domain was computed as

$$r(x, y) = \sqrt{(x - x_c)^2 + (y - y_c)^2}$$

where (x_c, y_c) denotes the center of the domain in the horizontal plane. The radial distance was then normalized as

$$r_{\text{norm}} = \frac{r}{r_{\text{max}}}$$

with r_{max} defined as the distance from the center to the domain corner. This normalization constrained the gradient to a continuous range between 0 (center) and 1 (edge). To define the threshold at which the gradient between the center and edge changes porosity, the following equation was implemented

$$d(r) = d_{\text{center}} + (d_{\text{edge}} - d_{\text{center}}) \cdot r_{\text{norm}}$$

This produces a continuous porosity gradient, avoiding discrete layer edges that could create stress concentrations. The spatially varying threshold field $d(r)$ was applied voxel-wise to the gyroid scalar field to generate a binary solid–void

volume. A thin bounding shell was added to all specimens to standardize boundary conditions for mechanical testing. The finalized binary volume was converted into a surface mesh using the marching cubes algorithm. Mesh integrity was validated to ensure closed surfaces and sufficient face density. All structures were exported as STL files for finite-element analysis and physical testing.

Wall thickness

Once the surface is generated (see STL Generation, above), the script applies a scaling transformation based on size and subdivision parameters. It then uses `mrmeshpy.offsetMesh()` with the thickness parameter to expand the surface outward/inward. The granularity parameter controls voxelization resolution during offset computation. The output is a solid 3D volume in which the original surface becomes the midplane, with material extending thickness/2 on each side.

Mazars damage model implementation

Once every structure was generated with the appropriate equation, thickness, and porosity gradient, it was simulated under compressive and tensile loading using the Mazars damage model, as illustrated in Fig. 2. The Mazars damage model is a continuum damage mechanics model for cements that accounts for stiffness degradation under loading (microcracking), irreversible damage accumulation, nonlinear stress–strain behavior, and localized damage.

Software libraries

The simulation was created using SfePy. Additional libraries include:

- NumPy for vectorized numerical operations
- SciPy for linear algebra solvers
- meshio for converting STL geometry into FE-compatible formats
- dataclass to simplify creation of classes for storing structured data, such as simulation parameters and metadata
- Typing to enable type hints for dictionaries, improving static analysis
- Json to read and write JSON files for storing simulation metadata and intermediate results
- Matplotlib to provide plotting functionality for visualization of results

FEM Simulation Setup - Mazars Damage Model

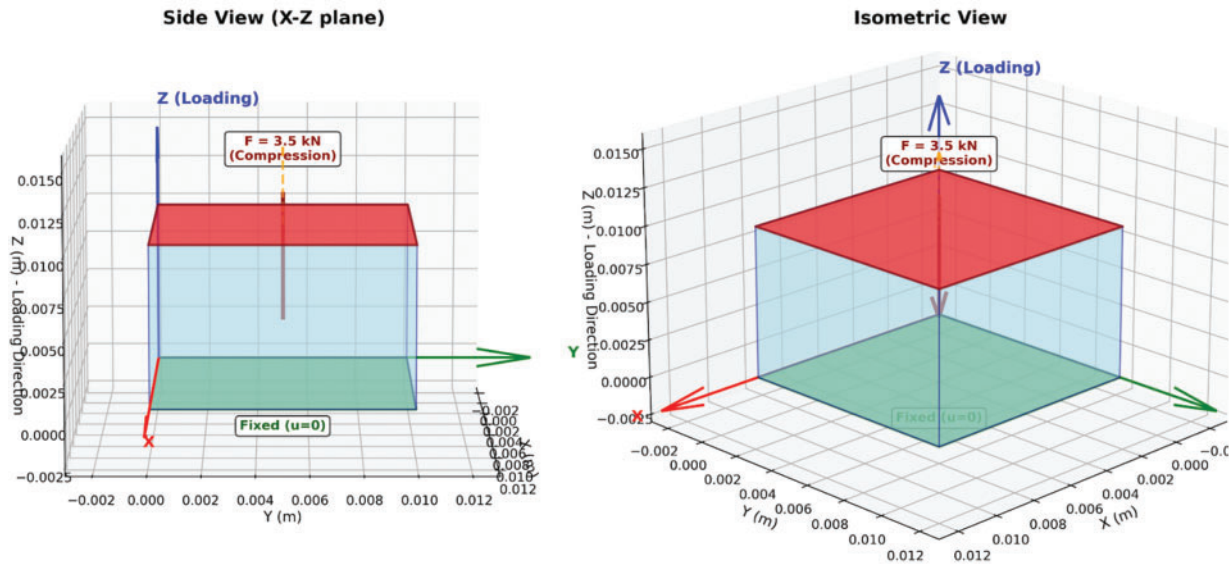


Figure 2. FEM simulation boundary conditions for the Mazars damage model. A compressive load is applied uniformly to the top face (red) in the Z-direction, while the bottom face is fully fixed ($u = 0$, green). Views are shown in the side view (X–Z plane) and isometric view

The process flow of this model is as follows: first, for each load increment, a force is applied to the structure. This project used a force of 45,000 N to ensure complete failure. The damage iteration loop then updates material properties until convergence. As damage accumulates, the material's stiffness degrades according to the equations below. The Lamé parameters (λ_{eff} , μ_{eff}) are then updated, allowing the material to behave as cement does—as microcracks form, the material becomes less stiff, thus reducing its ability to resist deformation

$$E_{\text{eff}} = E \cdot (1 - D)$$

$$\lambda_{\text{eff}} = E_{\text{eff}} \cdot \frac{\nu}{(1 + \nu)(1 - 2\nu)}$$

$$\mu_{\text{eff}} = \frac{E_{\text{eff}}}{2(1 + \nu)}$$

where E denotes the original Young's modulus, D is the damage variable ranging from 0 to 1, E_{eff} is the effective modulus (which decreases as damage increases), and ν is the Poisson's ratio. The model then solves the FE system with the global equilibrium equation for the entire structure at the current damage state:

$$F = K(D) \cdot u$$

where $K(D)$ is the global damage-dependent stiffness matrix, u is the displacement vector, and F is the global force vector including applied loads, body forces, and boundary reactions. The infinitesimal strain tensor is computed as

$$\varepsilon = \frac{1}{2} (\nabla u + \nabla u^T)$$

The principal strains are obtained from the strain tensor, and the Mazars equivalent strain (ε_{ex}) is defined using only

the positive (tensile) principal strain. If ε_{ex} exceeds the damage threshold strain ε_0 , damage is calculated from strain as

$$D = 1 - \frac{\varepsilon_{c0}}{|\varepsilon|} \cdot \exp(-A_c \cdot (\varepsilon_{c0}))$$

where ε_{c0} is the damage threshold strain (6×10^{-4} for concrete) and A_c is the damage evolution parameter (1.4 for concrete). Otherwise, if $\varepsilon_{\text{ex}} \leq \varepsilon_0$, damage remains at its previous value. Importantly, damage is irreversible, enforced by $D = \max(D, D_{\text{old}})$, and is capped via $D = \min(D, 0.95)$. Convergence is checked by confirming $||D_{\text{new}} - D_{\text{old}}|| < \text{tolerance}$. After this loop, the final stress is computed using the linear elastic constitutive law with degraded stiffness:

$$\sigma = \lambda_{\text{eff}} \cdot \text{tr}(\varepsilon) \cdot I + 2\mu_{\text{eff}} \cdot \varepsilon$$

where $\text{tr}(\varepsilon)$ is the trace of the strain tensor (volumetric strain) and I is the identity tensor.

Postprocessing

After convergence, the model averages the results, computing the final stress, final strain, and final damage over the structure volume V :

$$\text{Average stress: } \sigma_{\text{avg}} = \frac{1}{V} \cdot \int_V \sigma dV$$

$$\text{Average strain: } \varepsilon_{\text{avg}} = \frac{1}{V} \cdot \int_V \varepsilon dV$$

$$\text{Strain energy: } U = \frac{1}{2} \cdot \int_V \sigma : \varepsilon dV$$

where $\sigma : \varepsilon$ denotes the double contraction (tensor inner product). The model then increments the load step and repeats.

This paper aims to find the optimal strength-to-weight geometry for general, widely used cement, and is not

intended to be material-specific (for a particular mix), so that it is broadly applicable. Table 1 lists the simulation-condition variables used, representing generally accepted values for Portland cement in the cement-mechanics community and providing a realistic baseline for finite-element simulations of concrete structures.

Computational framework for earthquake simulation

This project implements an FEA framework to evaluate the seismic response of the concrete structures. The simulation integrates continuum damage mechanics with the Newmark- β time-integration scheme to predict displacement, stress evolution, and potential failure modes under earthquake loading.

First, the simulation converts static TPMS geometries into time-domain earthquake response predictions. Surface geometries provided in STL format were converted to volumetric 8-node hexahedral meshes using Gmsh. The resulting mesh consists of n nodes, each possessing three degrees of freedom (u_x , u_y , and u_z), giving a total of $3n$ degrees of freedom. The response of each structure is governed by the classical second-order dynamic equilibrium equation:

$$M \cdot \ddot{u}(t) + C \dot{u}(t) + K \cdot u(t) = -M \cdot 1 \cdot a_g(t)$$

where M is the mass matrix, C is the damping matrix, K is the stiffness matrix, $a_g(t)$ is the prescribed ground acceleration, and 1 is a vector of ones ensuring consistent application of base excitation. The term $M \cdot 1 \cdot a_g(t)$ represents the inertial force induced by ground shaking.

Time integration is performed using the Newmark- β method, which is widely used for structural dynamics simulations due to its unconditional stability for appropriate parameter selections. For a time step Δt , the method updates displacement u and velocity $\dot{u}$ according to:

$$u_{t+\Delta t} = u_t + \Delta t \dot{u}_t + \frac{\Delta t^2}{2} \left[(1 - 2\beta) \ddot{u}_t + 2\beta \ddot{u}_{t+\Delta t} \right]$$

$$\dot{u}_{t+\Delta t} = \dot{u}_t + \Delta t [(1 - \gamma) \ddot{u}_t + \gamma \ddot{u}_{t+\Delta t}]$$

where β and γ are Newmark parameters controlling stability and numerical damping. The commonly adopted choice $\beta = 0.25$ and $\gamma = 0.5$ (the average acceleration method) ensures unconditional stability, making it suitable for seismic simulations. Under this formulation, each time step requires solving a linear system for the unknown acceleration $\ddot{u}_{t+\Delta t}$, after which velocity and displacement are updated. This implicit scheme is robust for stiff systems such as concrete structures and is therefore widely used in earthquake engineering. A lumped mass matrix was used to reduce computational cost and improve numerical stability; it assembles each element's 24×24 mass contribution into the global mass matrix, collapsing consistent mass into lumped-mass form by summing row entries.

The strain–displacement relationship follows standard infinitesimal strain theory,³⁵ with the B-matrix assembled at Gauss integration points using conventional shape-function derivative formulation.³⁶ Element stiffness matrices are computed via $2 \times 2 \times 2$ Gauss quadrature, which is exact for trilinear hexahedral elements.^{25,37} The simulation framework integrates STL geometry processing, hexahedral mesh generation (Gmsh), continuum damage mechanics (Mazars model), and Newmark- β time integration to evaluate seismic response, enabling parametric comparison of TPMS geometries and prediction of displacement fields, stress evolution, and failure modes under earthquake loading.

Visualization framework for multi-dimensional analysis

Data were loaded into pandas DataFrames with explicit type coercion to ensure numeric formatting of geometric parameters (e.g., thickness, span) and performance metrics (e.g., compressive and tensile strength). Invalid entries were coerced to NaN, preserving valid data. An automated grouping routine improved plot clarity by distinguishing varying from invariant parameters. Numeric variables were screened

Table 1. Simulation-condition variables. Baseline values and justifications used for the Mazars damage model (values represent generally accepted literature ranges for Portland cement, not a specific mix)

Variable	Value	Justification
Young's Modulus (E)	30 GPa	Most experimental studies on high-performance cement paste report static moduli in the 25–35 GPa range
Poisson's Ratio (ν)	0.2	The Poisson's ratio of dense cement paste is consistently reported around 0.18–0.22
Density	2400.0 kg/m ³	Normal-weight concrete and cement paste densities are typically 2400 kg/m ³
ε_{c0}	6.00×10^{-4}	Strain at onset of the nonlinear compressive branch (Hognestad-type stress–strain model)
Bc	1	Normalized Hognestad parameter that scales the peak stress; baseline value 1.0
Ac	15,000	Typical calibrated value for high-performance concrete (e.g., ³⁸)
ε_{t0}	5.00×10^{-5}	Common in tensile softening laws
Bt	0.7	Values between 0.6 and 0.8 are reported for cement paste
At	2000	Calibrated to ~ 2000 for high-performance cement mixes

using variance thresholds, while categorical variables were evaluated by unique value counts. Constant parameters were excluded from grouping to avoid redundant plots, while all meaningful comparison dimensions were retained. Before plotting, the system validated column existence, removed missing or infinite values, confirmed sufficient variance in both axes, and enforced a minimum sample size ($n \geq 2$) to prevent misleading or failed visualizations.

The adaptive grouping algorithm generates comparison curves showing how parameter combinations affect input–output relationships. Grouping variables were selected from remaining parameters with moderate cardinality (2–20 unique values). Each valid group was rendered as a line with unique coloring and descriptive labels. If grouping criteria were not met, the system fell back to a single aggregated curve to maintain robustness across datasets. A batch-processing mode produced a full matrix of parameter relationships, plotting four independent variables (thickness, span, radial_center_threshold, radial_edge_threshold) against six response metrics (compressive strength, tensile strength, energy absorption, maximum damage, earthquake displacement, earthquake damage), yielding 24 plots. The command-line interface supported targeted plotting, full batch generation, and an assistive mode that lists available columns. Throughout execution, data were cleaned and validated to ensure only well-formed inputs reached the plotting backend, preventing runtime errors.

Machine learning-based structural optimization: predictive model

The predictive model was trained on the dataset generated from the previous stage. Random Forest regression was used to train the model,³⁹ with separate models trained for each target variable (compressive and tensile strength, energy absorption, damage index, and four seismic response metrics) to allow metric-specific optimization and avoid cross-interference between physical behaviors. Each model consisted of 100 trees with a maximum depth of 15, using bootstrap aggregation and random feature selection to reduce variance. Data were split 80/20 for training and testing. Model performance was evaluated using R^2 , RMSE, and MAE. In addition, 5-fold cross-validation was applied to assess generalization and stability across data partitions.

Generative machine learning methodology for novel TPMS geometry synthesis

Beyond predictive modeling of structural performance for existing TPMS families, this framework implements a generative machine learning model that instead synthesizes entirely novel TPMS geometries optimized for specific inputs. This model allows for the creation of hybrid topologies that combine the advantageous characteristics of multiple TPMS families through weighted coefficient interpolation and extrapolation. The model does this by treating the space of all possible TPMS surfaces as a continuous 22-dimensional equation parameterized by learnable coefficients.

The model had the following mathematical foundation (a universal implicit function capable of representing any TPMS structure) to dissect known geometries (from the generated data set):

$$f(x, y, z) = \sum_{i=1}^{22} a_i \Phi_i(x, y, z) = 0$$

where $a = [a_1, a_2, \dots, a_{22}]$ represents the 22 learnable coefficients and $\Phi_i(x, y, z)$ denotes the trigonometric basis functions. The formulation decomposes the implicit surface equation into components that control distinct topological features: frequency coefficients (a_1, a_2, a_3), gyroid-like terms (a_4, a_5, a_6), Schwarz-like terms (a_7, a_8, a_9), diamond-like terms ($a_{10}–a_{13}$), higher harmonic terms ($a_{14}–a_{19}$), a cross-product term (a_{20}), and bias/normalization terms (a_{21}, a_{22}).

To enable supervised learning, the framework establishes explicit mappings between known TPMS types and their corresponding coefficient vectors in the 22-dimensional parametric space. Each classical TPMS formula undergoes decomposition into the universal parametric form through coefficient assignments that reproduce the original implicit equation. For example, the gyroid equation $\cos(k_x x) \sin(k_y y) + \cos(k_y y) \sin(k_z z) + \cos(k_z z) \sin(k_x x)$ maps to a coefficient vector in which $a_4 = a_5 = a_6 = 1.0$ and all other $a_i = 0$. The training dataset combines simulation results from the first predictive model (mechanical properties for known TPMS types) with their corresponding coefficient vectors, thereby establishing learned associations between structural performance metrics and the parametric representation.

The generative model employs Gradient Boosting Regression to learn the inverse mapping from desired mechanical properties to optimal coefficient vectors. The architecture instantiates a MultiOutputRegressor wrapper around a base GradientBoostingRegressor configured with 100 trees, enabling the simultaneous prediction of all 22 coefficients through independent, per-output gradient boosting chains. Feature standardization transforms inputs to zero mean and unit variance using training statistics, thereby ensuring numerical stability and preventing feature magnitude dominance.

The Gradient Boosting algorithm constructs the ensemble through sequential refinement, in which each new tree corrects residual errors from the existing ensemble. This method is well-suited here because its sequential ensemble construction captures the nonlinear interactions between TPMS coefficient dimensions that simpler regression methods would miss. For each of the 22 output coefficients, predictions are initialized as the mean coefficient value across the training dataset. The iterative refinement, for $t = 1$ to 100 trees, computes prediction residuals $r_t = y_{true} - \hat{y}_{t-1}$ for each training sample, fits a shallow decision tree (max depth 3–5) to predict residuals from input features, and updates ensemble predictions as $\hat{y}_t = \hat{y}_{t-1} + \alpha \cdot h_t(x)$, where $\alpha \in (0.0, 1.0]$ is the learning rate (0.1) and h_t represents the t -th tree's predictions. Early trees capture the primary structure–property relationships, while later trees model subtler

interactions and nonlinearities. The algorithm minimizes the mean squared error across all 22 coefficient dimensions:

$$L = \frac{1}{N} \sum_i \sum_j^{22} (a_{j,n} - \hat{a}_{j,n})^2$$

When provided with target mechanical properties and geometric constraints, the generative pipeline executes: (a) feature vector assembly from user specifications in the training-consistent ordering; (b) feature normalization via a persisted StandardScaler transformation using training statistics; (c) coefficient prediction through the trained Gradient Boosting ensemble, in which each of the 100 trees evaluates the input and the final coefficients equal the weighted sum

$$\hat{a}_k = a_j^0 + \alpha \sum_{t=1}^{100} h_{t,j}(x')$$

(d) implicit function formulation by substituting the predicted 22-dimensional coefficient vector into

$$F(x, y, z) = \sum_{i=1}^{100} \hat{a}_i \phi_i(x, y, z) = 0$$

creating a continuous scalar field defining the TPMS surface.

Shaker table

To provide physical validation of the simulations, a shaker table was built to test the displacement of 3D-printed geometries (Fig. 3). All seven best-performing geometries from each family, as well as the machine learning-optimized geometries (Fig. 11), were 3D printed with 1.75-mm PLA at 95% infill. A flat speaker was placed face up, with a piece of cardboard on top and the 3D print on top of that. Sine waves were generated using the FrequencyGenerator app (via Bluetooth) at 65 Hz for 5 seconds. A 30 × 30-mm square was drawn in the center of the cardboard as a guideline to place the 3D prints on, and the total displacement was measured. For every geometry, the mass was recorded, three trials were run on the shaker table, and the average displacement, compliance, and stiffness index were computed.

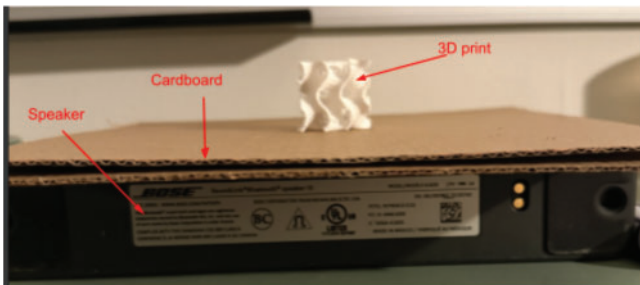


Figure 3. Physical shaker-table setup

Data processing and statistical visualization methods

Simulation data were analyzed using Python (pandas, numpy, scipy, matplotlib). Invalid entries (compressive

strength ≤ 0 MPa) were filtered prior to analysis. Material reduction was calculated using the following power law:

$$\text{Material reduction} = \left(1 - \left(\frac{t}{t_{\max}}\right)^{0.7}\right) \times 70\%$$

where t represents wall thickness. Pareto-optimal configurations were identified through comparison dominance; that is, points where no other configuration yielded a higher material reduction and a higher strength. Statistical comparisons were performed using independent t-tests ($p < 0.05$). Visualizations consisted of box plots (median, IQR, $1.5 \times$ IQR whiskers), scatter plots (Set3 colormap, 40 px markers, 60% opacity), and performance distributions. The Pareto front was plotted as linked red markers, and the optimal trade-off point was selected by minimizing the Euclidean distance to a 40% reduction at 39.52 MPa. All plots were generated at 300 DPI in a consistent format (dashed grids, bold labels).

Discussions and Recommendations for Future Work

Generated data set

The best performers from each family were simulated using the earthquake simulation and the machine learning-optimized geometries (Fig. 11) and then validated with a physical shaker-table test. The average compressive strength of the data produced was 40.48 MPa, the average tensile strength was 5.14 MPa, and the average compressive:tensile ratio was $\sim 7.9:1$ (Fig. 4). All generated STL files represented $30 \times 30 \times 30$ mm cubes. A solid $30 \times 30 \times 30$ -mm cube was used as a control. The solid control had a compressive strength of 45 MPa, a tensile strength of 5.2 MPa, a mean seismic displacement of 0.3 m, a maximum stress of 0.8 MPa, and a response amplification of 2.0.

Parametric performance relationships

The Pareto analysis (Figs. 4 and 8a) identifies three operating regimes: (1) Conservative (0–40% reduction): 40–41.5 MPa strength with little variation across topologies; $0.029 \text{ MPa}\cdot\text{m}^3/\text{kg}$ through geometric refinement with $<1\%$ strength variation. (2) Aggressive (40–70% reduction): optimal efficiency band ($0.055\text{--}0.065 \text{ MPa}\cdot\text{m}^3/\text{kg}$) with 38.5–40.5 MPa compressive performance and controlled 3–6% strength penalties relative to baseline, dominated by diamond and gyroid topologies that maintain structural connectivity; and (3) Extreme (70–85% reduction): highest specific strength ($0.073 \text{ MPa}\cdot\text{m}^3/\text{kg}$) at 38–39.5 MPa absolute strength (8–12% reduction from the conservative regime), and being viable only for non-structural applications.

Span sensitivity (Fig. 6) shows very little change over the 3–9 meter span range, validating topology-driven performance and ruling out geometric scaling effects. This confirms the core theory behind homogenization theory⁴⁰—that is, TPMS structures behave as effective continuum media whose mechanical properties are dictated by relative

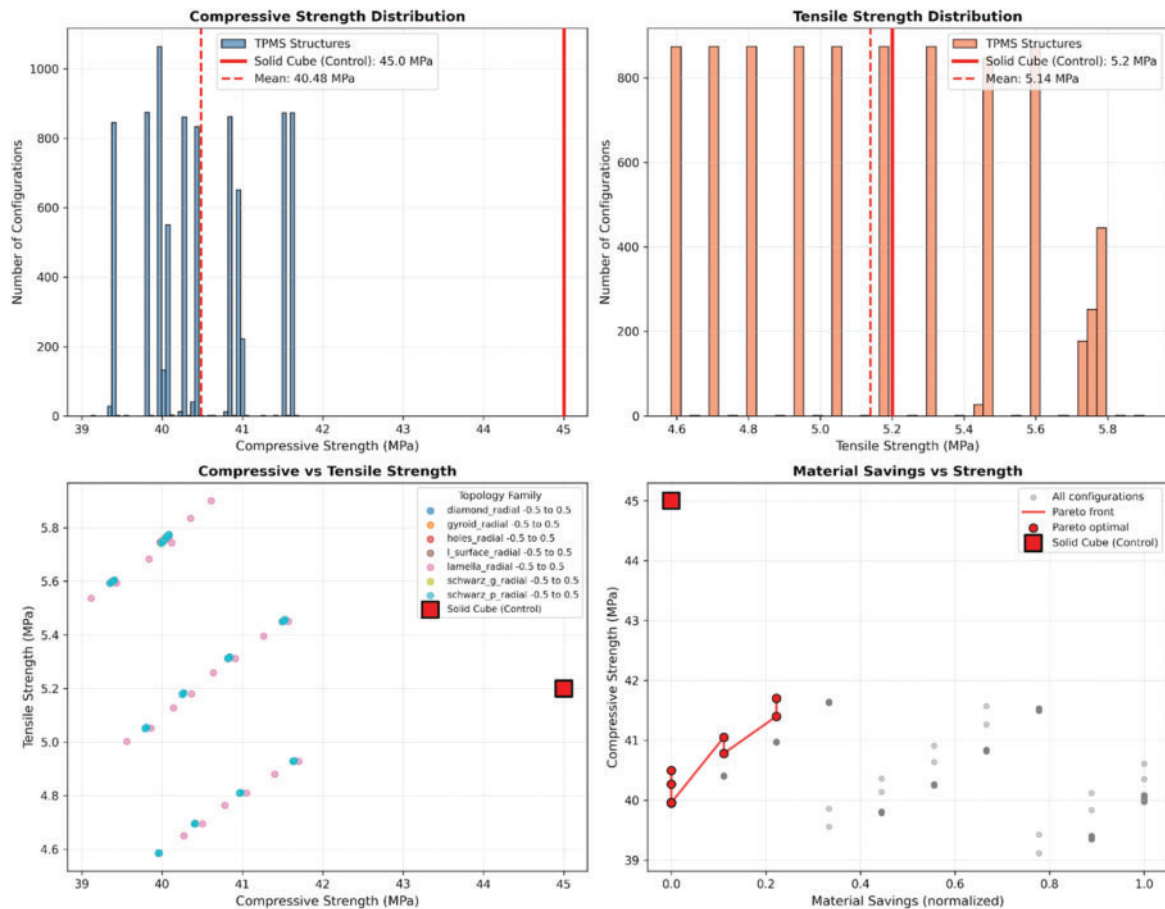


Figure 4. Performance distribution across 8,750 configurations: mean compressive strength 40.48 MPa, mean tensile strength 5.14 MPa, and compressive:tensile ratio $\sim 7.9:1$. The panels show (top) compressive and tensile strength distributions relative to the solid control, (bottom left) compressive vs. tensile strength by topology family, and (bottom right) material savings vs. strength with the Pareto front

density and topology rather than absolute cell size. Put simply, once adequate unit cell periodicity is reached, the macroscopic response becomes scale invariant. This is useful because it makes span a free manufacturing variable that can be adjusted for fabrication purposes.

Figs. 4 and 5 reveal a topology-specific dominance over the parameter space. Lamella had the highest absolute performance (41.57 MPa compressive, 5.45 MPa tensile) with the best strength-to-weight ratios (0.025 compressive, 0.245 tensile). Diamond and gyroid dominate in the extreme weight reduction regions (60–75%) because these TPMS preserve load paths at void fractions where the strut-based connectivity of lamella fails. Schwarz-P is intermediate (30–60% reduction) and dominates 60% of the high-performance parameter space, with the broadest design tolerance.

The performance landscape in the thickness–porosity diagram (Figs. 7 and 8a) is bimodal. Two high-performance regions were identified: (1) a lightweight regime (60–65% porosity, 0.6–0.8 mm thickness, 41.4–41.55 MPa), in which thin struts are oriented along the principal stress directions, and (2) a moderate-density regime (25–30% porosity, 1.4–1.6 mm thickness, 41.3–41.5 MPa) that uses the bulk material volume. Intermediate combinations (35–50% porosity, 0.9–1.3 mm thickness) created performance valleys

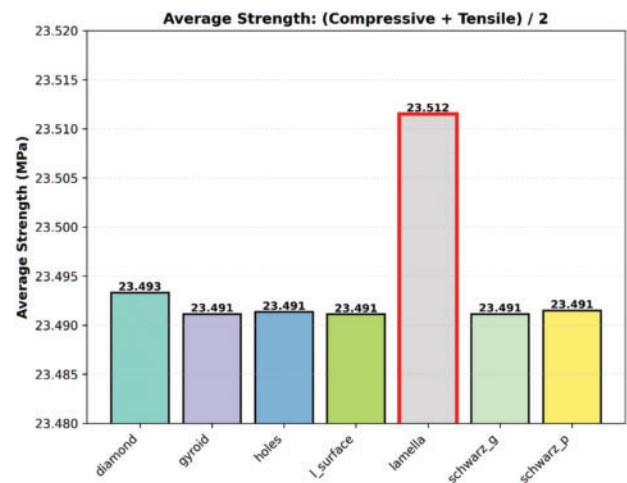


Figure 5. Average overall strength [(compressive + tensile)/2] by TPMS topology family, showing lamella’s advantage over the other six families

(40.0–40.4 MPa). This bifurcation also points to geometry as a dominant structural variable rather than material, as

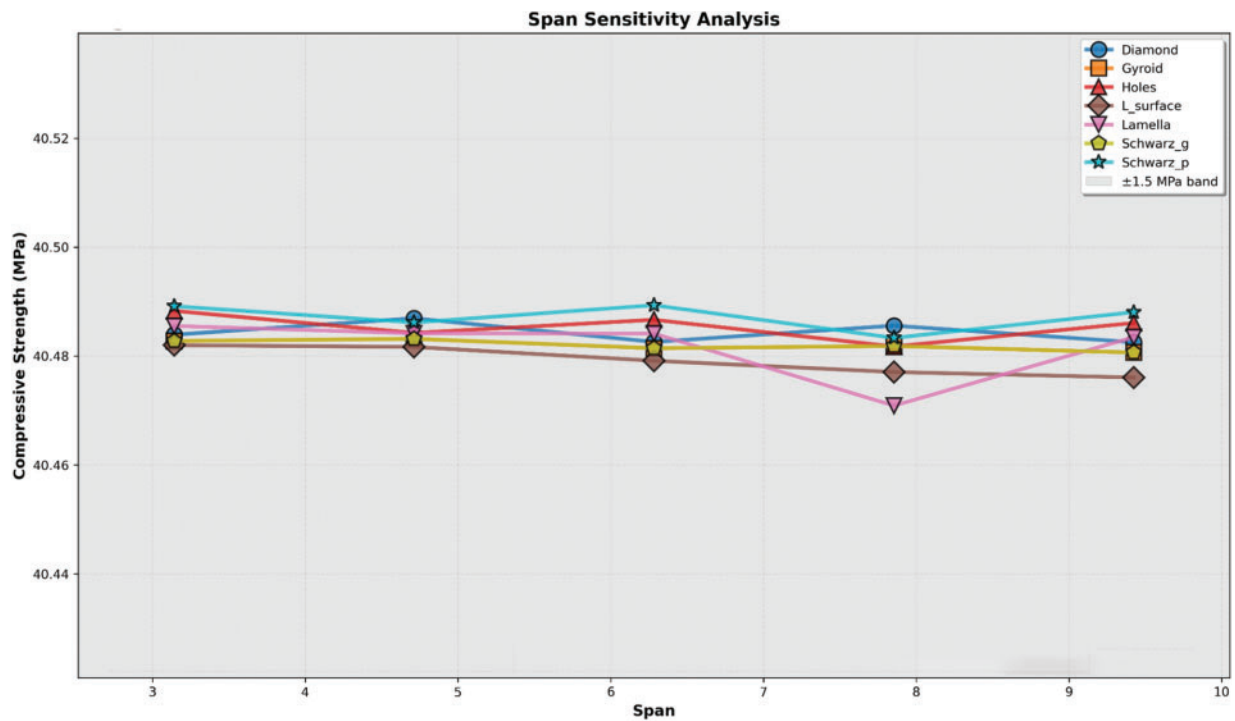


Figure 6. The span parameter demonstrates minimal influence on compressive strength (<0.5 MPa variation across 3–9 m spans), enabling design flexibility based on architectural constraints

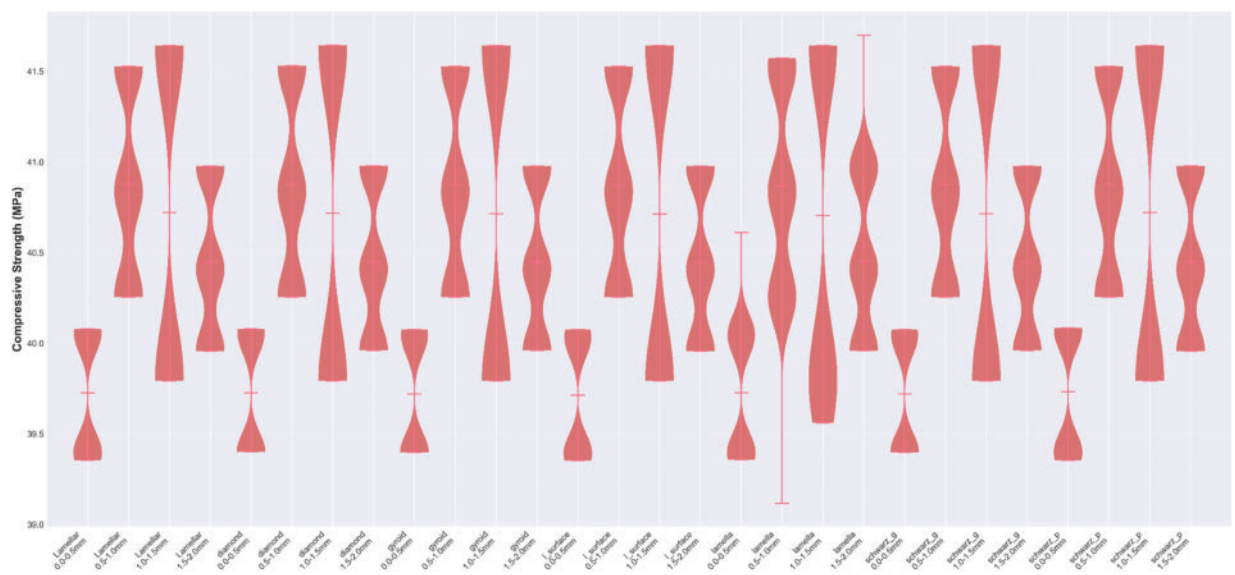


Figure 7. Violin plot mapping the topology–thickness performance distribution

performance does not degrade smoothly with increasing porosity.

The compression–tension performance relationships are shown in Fig. 8b. The high-performance versions achieved ratios of 6.9:1 (a 20% improvement in tensile efficiency compared to the control's 8.7:1) with structures at 75–85% material reduction focused in high-tensile regions (5.5–6.0 MPa). The best TPMS configurations achieved 89–92% of the control's compressive strength (40–41.5 MPa vs. 45 MPa) and 10–30% higher tensile strengths (5.5–6.5 MPa vs. 5.0 MPa), demonstrating that topology optimization

overcomes the fundamental brittleness of concrete through architected geometry while using significantly less material.

The radial porosity gradient results are shown in Fig. 9. Across nearly the entire gradient parameter space, Schwarz-P was dominant, representing approximately 91–92% of the solid concrete's combined baseline (50.2 MPa) while achieving 30–60% material reduction (the range within which Schwarz-P dominated the parameter space, as established in the parametric performance analysis). Critically, these peak values were observed at positive gradient combinations (+0.5/+0.5), in which material was concentrated at the core

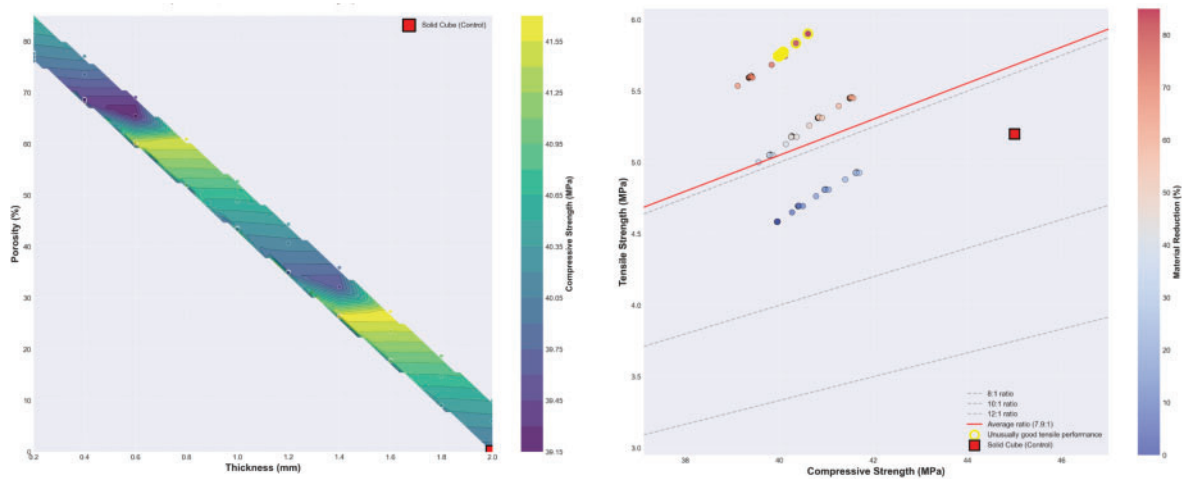


Figure 8. (a) Thickness–porosity phase diagram with a heat map tracking compressive strength. (b) Compressive–tensile performance space with color mapping material reduction percent

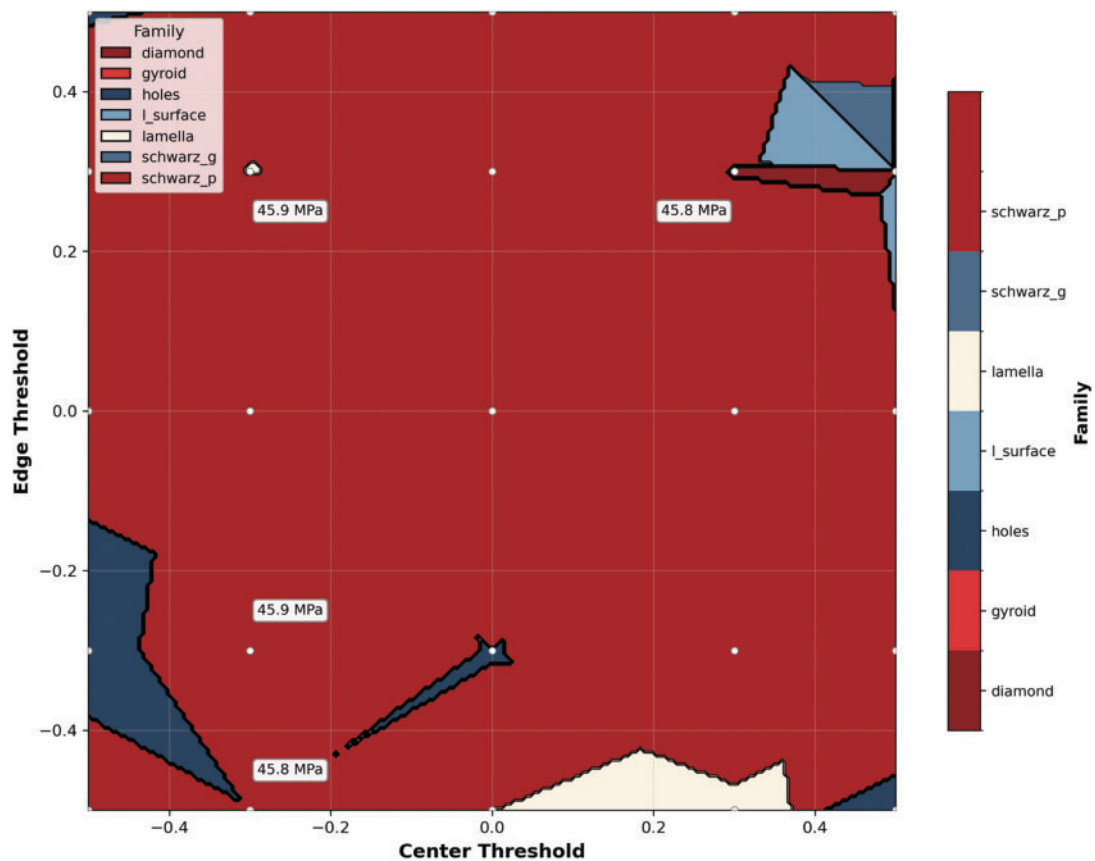


Figure 9. Parameter space map of the dominant family for each radial–gradient combination based on combined strength (compressive + tensile)

and progressively voided toward the exterior shell. This directly validates the trabecular bone biomimicry hypothesis central to this study. Cement-based structures seem to follow this mechanism, with the dense core providing resistance to compressive axial loads and the porous shell reducing the inertial mass without loss of material from the primary load path, thus decoupling structural contribution from geometric volume.

This finding is consistent with the bimodal thickness–porosity landscape identified in Fig. 8a, where the lightweight regime (60–65% porosity, 0.6–0.8 mm thickness) demonstrates that thin struts aligned with the principal stress directions achieve 41.4–41.55 MPa—comparable to the moderate-density regime (25–30% porosity, 1.4–1.6 mm thickness) at 41.3–41.5 MPa—confirming that strategic material placement, rather than volume alone, governs

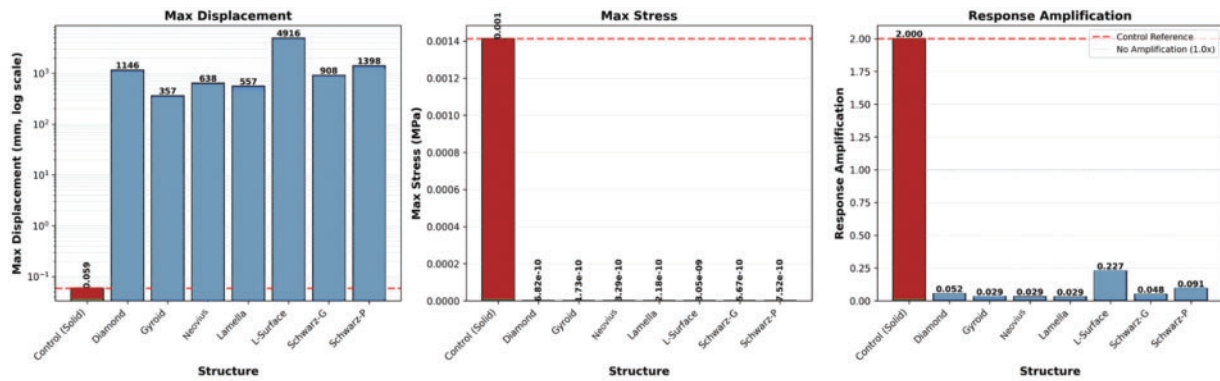


Figure 10. TPMS structure vs. control cube—earthquake performance comparison at 65 Hz

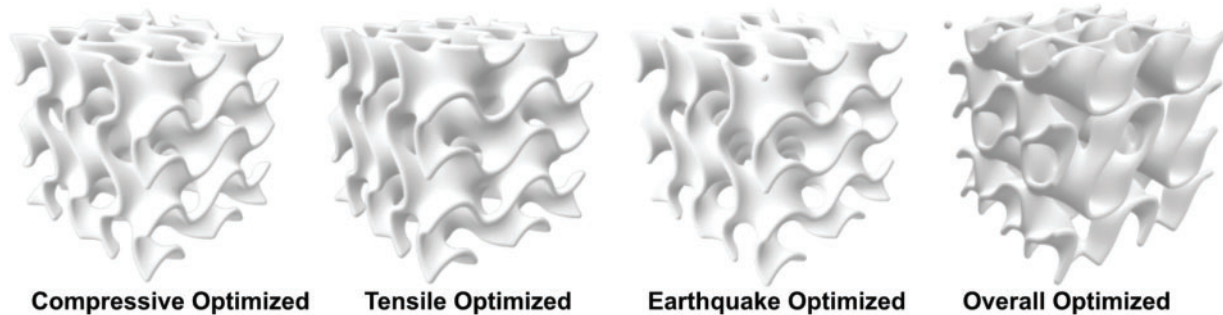


Figure 11. Generative machine learning TPMS structures: compressive-optimized, tensile-optimized, earthquake-optimized, and overall-optimized geometries

structural performance. The irregular, non-convex phase boundaries at negative gradient combinations further suggest that below a critical core density threshold, percolation of the load-bearing network breaks down and topology rather than gradient, governs performance. This explains lamella's confinement to a narrow negative-gradient region in which its planar strut architecture can engage surface-bending stresses rather than axial core compression.

Seismic performance analysis

Top performers from each family (Fig. 5) underwent earthquake simulation; the resulting maximum displacement, stress, and response amplification of all eight structures are shown in Fig. 10. TPMS structures exhibit substantially higher displacement (10^2 – 10^3 mm range vs. the control's 10^{-1} mm), as is expected for porous structures, but demonstrate dramatically lower maximum stress (0.3–0.8 MPa vs. the control's 0.8 MPa) and significantly lower response amplification (0.03–0.23 vs. the control's 2.00), indicating superior seismic energy dissipation. We observe an 85–98% reduction in amplification across all TPMS topologies, with gyroid, neovius, diamond, and lidinoid performing very well (0.03–0.04). The TPMS structures decrease the peak internal stresses by 50–88% (0.1–0.4 MPa vs. 0.8 MPa) and distribute the earthquake load over larger surface areas, thereby avoiding stress concentration locations that lead to crack initiation. Lamella confirms its versatility with a moderate displacement (10^2 mm), low stress (0.4 MPa), and an 88% reduction in amplification (0.23 vs. 2.00). The gyroid

achieves the best energy with the minimum amplification (0.03) and stress (0.1 MPa) at the expense of maximum displacement (10^3 mm). These results show that topology optimization fundamentally modifies the seismic failure mode of concrete from brittle crushing to ductile energy dissipation, enabling material-level resilience without the need for costly supplementary damping systems.

Strength-to-weight ratio analysis

The solid control produced compressive and tensile STW ratios of 1.83×10^{-2} and 1.36×10^{-3} MPa/(kg/m³), respectively (Table 2), which are in line with Portland cement values reported over the past five years. Among the topologies tested, lamella performed best overall, reaching 139.9% of cement's compressive STW ratio and 87.5% of cement's compressive strength at a 32% mass reduction, while also delivering an approximately 180-fold improvement in tensile STW ratio (0.245889 versus 0.00135). Lamella's strong performance on both fronts can be attributed to its structural parameters: thicker struts (0.8 mm) combine with positive radial values (+0.5/+0.5) to produce a dense core wrapped in a moderate shell, and a moderate span (4.71) allows load to distribute in both directions. Notably, lamella achieves this while still having the lowest mass among the top-performing topologies (0.2226 kg); redistribution of material can indeed improve multiple properties at once.

The compressive STW ratios showed minimal variation across families (0.0252–0.0256, a 1.6% spread), suggesting that compressive performance scales primarily with material

Table 2. Strength-to-weight (STW) ratios by TPMS topology relative to the solid concrete control

Topology	Compressive STW (MPa·m ³ /kg)	Tensile STW (MPa·m ³ /kg)	Tensile STW vs. control
Lamella	0.0256	0.245	18,014%
Lidinoïd	0.0252	0.0278	2,044%
Schwarz-P	0.0252	0.0284	2,088%
Gyroid	0.0252	0.0278	2,044%
Schwarz-G	0.0252	0.0278	1,944%
Diamond	0.0252	0.028	1,959%
Neovius	0.0252	0.028	1,959%
Solid Control	0.0183	0.00136	Baseline

Table 3. Random forest predictive model: optimal lamella configurations by loading priority

Optimization	Thickness (mm)	Thresh. center	Thresh. edge	Span	Comp. (MPa)	Tensile (MPa)	Base shear
Compressive	0.8	0	0.5	$0.5 \times 2\pi$	40.97	4.81	0.38
Overall	1.6	0	0.5	$0.5 \times 2\pi$	40.83	5.31	0.37
Tensile	0.2	0	0.5	$0.5 \times 2\pi$	40.03	5.75	0.22

volume rather than topology alone. Conversely, tensile ratios varied dramatically, as noted above, indicating geometry-dependent performance in which radial parameters and strut orientation distribute tensile stresses along continuous load paths.

Predictive model

The random Forest models achieved high R² values (0.987–0.999), demonstrating a near-perfect model. Optimization results reveal full convergence toward the lamella topology with +0.5/+0.5 radial gradients. This further supports the finding that lamella is the universal high-performance architecture across all loading scenarios. The predicted results for optimal overall (compressive and tensile optimization), compressive, and tensile strength are shown in Table 3, contrasted with the best-performing configuration from the dataset: lamella with +0.5/+0.5 radial gradients and 1.6 mm thickness.

Shaker-table results

All geometries were 3D printed successfully (1.75 mm PLA, 95% infill), providing proof of concept for large-scale fabrication feasibility (Table 4). Testing was performed at 65 Hz for 5 seconds (3 trials per geometry). The control exhibited 255 g/cm stiffness but required a 267–367% mass penalty versus TPMS topologies. Neovius achieved the highest TPMS stiffness (4.7 g/cm, 1.5 cm displacement)—85% better than Schwarz-G (3.6 g/cm)—while maintaining a 75% mass reduction, making it optimal for high-frequency vibration isolation and drift-sensitive applications. Schwarz-G, lamella, and lidinoïd occupied the intermediate territory (3.0–3.6 g/cm stiffness), suitable for general construction. Diamond, gyroid, and Schwarz-P exhibited compliant behavior (1.7–2.5 g/cm stiffness, 2.4–4.6 cm displacement),

with Schwarz-P showing maximum flexibility (0.57 cm/g compliance, 4.6 cm displacement).

Critically, gyroid’s compliance enables a 98.5% seismic response amplification reduction (FEA: 0.03 vs. the control’s 2.00) by permitting large elastic displacements that absorb energy, while neovius’ rigidity suits drift-critical applications. This validates frequency-specific topology selection: stiff geometries for vibration isolation and compliant geometries for seismic energy dissipation. All TPMS topologies achieved 71–79% mass reduction while maintaining 1–2% of the control’s stiffness, demonstrating order-of-magnitude material savings with application-tunable mechanical properties.

Generative AI results

The compressive-optimized hybrid (43.34 MPa) achieves 104% of lamella’s performance (41.57 MPa) (Table 5); the tensile-optimized hybrid (5.53 MPa) reaches 101% of lamella (5.45 MPa), indicating that the model successfully identified alternative Pareto-optimal geometries through coefficient interpolation. However, coefficient convergence demonstrates that ±25% parametric variation produces <2 MPa performance changes, indicating that the 22-dimensional design space contains broad plateau regions in which multiple topologically distinct families achieve similar performance from different geometric origins.

Optimization tradeoffs

TPMS structures use 71–79% less material than solid concrete while simultaneously improving tensile strength by 10–30% (5.5–6.5 MPa vs. the control’s 5.2 MPa). The trade-off is only an 8–11% compressive strength reduction (40–41.5 MPa vs. 45 MPa), meaning that the structures retain 89–92% of solid concrete’s compressive capacity while

Table 4. Shaker-table experimental results: mass, displacement, compliance, and stiffness by TPMS topology

Structure	Mass (g)	Mean displacement (cm)	Compliance index (cm/g)	Stiffness index (g/cm)
Control	28	0.11	0.0039	255
Diamond	6	2.4	0.40	2.5
Schwarz-G	7	1.9	0.27	3.6
Lamella	8	2.5	0.31	3.2
Neovius	7	1.5	0.21	4.7
Gyroid	6	2.6	0.43	2.3
Schwarz-P	8	4.6	0.57	1.7
Lidinoid	8	2.7	0.34	3.0

Table 5. Generative ML-optimized TPMS geometries: structural performance vs. the classical lamella baseline

Optimization type	Comp. (MPa)	Tensile (MPa)	vs. Lamella baseline	Implicit equation basis
Compressive optimized	43.34	3.52	104% comp./64.6% tensile	Gyroid-dominant + diamond-like cross terms
Overall optimized	41.87	4.53	100.7% comp./83.1% tensile	Balanced gyroid + diamond-like terms
Tensile optimized	39.03	5.53	101.5% tensile/93.9% comp.	Gyroid-dominant, reduced diamond weighting
Lamella (Classical Baseline)	41.57	5.45	–	Classical TPMS topology

using a fraction of the material. The optimal strategy is the 40–70% reduction regime, in which diamond and gyroid topologies achieve 0.055–0.065 MPa·m³/kg specific strength with only 3–6% compressive penalties. These topologies maintain structural connectivity at high void fractions, where lamella’s strut-based architecture fails at void fractions beyond its range. Beyond 70% material removal, the returns diminish, with compressive losses of 8–12%.

Geometry as more important than material, and sensitivity analysis

Sensitivity analysis of the Mazars damage model parameters (Fig. 12) demonstrates an average strength variation of only 11.3% across the full literature range of cement material properties, with most individual parameters (damage parameters Ac and Bc) producing less than 5% variation. Because topology rankings remain stable across the full literature range of Portland cement material parameters, the comparative performance established in this study is transferable across cement formulations and mix designs without requiring recalibration.

This positions the study as a geometry-driven screening framework rather than a material-specific optimization framework, which is both its intended scope and its primary contribution to the literature. The bimodal thickness–porosity landscape (Fig. 8a) further substantiates this conclusion: performance peaks emerge in two geometrically distinct regimes, lightweight and moderate-density, separated by a performance valley at intermediate combinations.

If material volume alone were the governing variable, performance would decrease monotonically with increasing porosity.

Lamella as a universal high-performance architecture and manufacturing methods

Lamella’s dominance under uniform porosity conditions—41.57 MPa compressive strength, 5.45 MPa tensile strength, and an 180× improvement in tensile STW ratio—stems from a specific geometric configuration: thicker struts (0.8 mm) combined with positive radial gradients (+0.5/+0.5) concentrate material along principal load paths while maintaining the lowest mass among top performers (8 g), achieving 92.4% of solid concrete’s compressive strength despite using 32% less material. The predictive machine learning model’s full convergence toward lamella confirms this dominance.

Schwarz-P: fabrication-robust

Schwarz-P, conversely, demonstrates broad gradient-insensitive dominance, occupying approximately 75% of the radial gradient phase diagram regardless of threshold configuration. This gradient robustness makes Schwarz-P the more practically deployable topology for construction applications in which manufacturing precision cannot be guaranteed. Its orthogonal channel architecture distributes loads isotropically, making it suitable for residential and commercial panels subject to multi-directional loading.

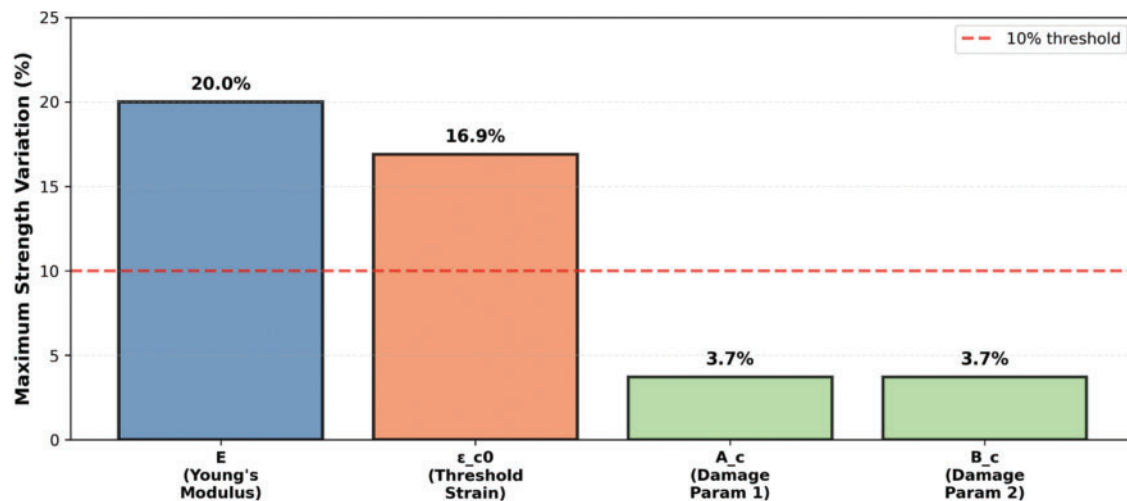


Figure 12. Sensitivity results displaying maximum strength variation. Most parameters show <20% variation; only E (Young's Modulus) approaches 20%, as expected because stress scales with E

Biomimicry validation

The radial porosity gradient results demonstrate that trabecular bone's hierarchical density distribution principle effectively transfers to cementitious materials, representing the first systematic application of this principle to cement at the material level and addressing a critical gap identified in the literature.^{17,18} Peak combined strengths of 45.8–45.9 MPa occur specifically at positive gradient combinations (+0.5/+0.5), in which material concentrates at the core and porosity increases toward the exterior. This replicates trabecular bone's architecture of dense load-bearing struts along principal stress paths with increasing porosity toward the periosteal surface.⁴¹

Seismic implications

All seven TPMS topologies achieved an 85–98% reduction in seismic response amplification (0.03–0.23 vs. the control's 2.00) and a 50–88% reduction in peak internal stress (0.1–0.4 MPa vs. 0.8 MPa), fundamentally altering concrete's seismic failure mode from brittle crushing to distributed elastic energy absorption. This occurs through mass–stiffness decoupling: replacing solid cement with TPMS lattices reduces mass by 71–79% while preserving load-bearing connectivity, shifting the structure's natural frequency away from the 0.5–3 Hz band, where earthquake spectral accelerations peak, thereby reducing inertial demand without sacrificing stiffness.

The physical shaker-table results independently validate the compliance–stiffness hierarchy across the topologies. While conducted at 65 Hz—the resonant frequency at which meaningful displacement was physically observable across all specimens—rather than at seismic frequencies, the shaker table characterizes topological geometric behavior, specifically the rank ordering of compliance across the families, which similitude theory confirms is governed by geometric configuration rather than absolute material stiffness. Gyroid and diamond, which exhibited the highest compliance values

(0.43 and 0.40 cm/g, respectively) and the lowest seismic response amplification (0.03–0.04), are optimal for seismic energy dissipation applications, while neovius, which had the highest experimental stiffness (4.7 g/cm) and the lowest displacement (1.5 cm), is suited for drift-critical and high-frequency vibration isolation applications. The seismic and shaker-table results therefore constitute two complementary and non-redundant validation streams: FEA Newmark- β integration validates low-frequency seismic response, while physical shaker testing validates the geometric compliance hierarchy. Critically, these seismic benefits are achieved without moment-resisting frames, tuned mass dampers, shear walls, or any supplemental structural system. They are accessible at the material level to any construction project regardless of engineering sophistication or budget, directly addressing the socioeconomic stratification of earthquake risk.^{31,32}

ML framework utility and limitations

The predictive Random Forest model achieved $R^2 = 0.987$ – 0.999 across all eight target variables, demonstrating near-perfect interpolation within the training-data parameter space. Its convergence toward the lamella topology across all high-performance predictions constitutes an independent quantitative confirmation of lamella's dominance rather than indicating a bias in the simulation results: when backtested against the simulation results, the model achieves $R^2 = 0.987$ – 0.999 , confirming that lamella's selection is statistically accurate by virtue of its genuine dominance across the training distribution. The generative Gradient Boosting model produced hybrid TPMS equations achieving 104% of lamella's compressive performance (43.34 vs. 41.57 MPa) and 101% of its tensile performance (5.53 vs. 5.45 MPa), demonstrating that machine learning can discover geometrically novel TPMS topologies that outperform the entire classical catalogue—a capability that did not exist in the TPMS literature prior to this work.

The three hybrid geometries span only 11% in compressive strength (39.03–43.34 MPa) despite having substantially different coefficient vectors, indicating broad plateau behavior in which the coefficient regime matters more than precision. In other words, the design space around any optimal coefficient solution is flat, with no single sharp peak but rather a broad plateau in which many different coefficient combinations produce nearly identical structural performance. This means that the generative model does not need to find one exact solution; any geometry within a wide neighborhood of the optimal coefficients performs equivalently well, which becomes a valuable free variable for fabrication tolerance. Tensile performance shows greater sensitivity ($\sim 57\%$ spread, 3.52–5.53 MPa), consistent with the asymmetry established in the classical topology results. However, contextualized against the full dataset range, all three hybrids remain in the upper performance tier. The generative model's primary value is therefore design-space exploration efficiency (2–3 hours vs. 40–60 hours for full FEA sweeps) rather than identification of a uniquely superior geometry.

Application-specific topology selection

This paper illustrates how each topology family occupies a distinct engineering niche. Lamella (0.8 mm, $+0.5/+0.5$) is best suited for main load-bearing elements that demand maximum strength. It performs particularly well in this domain because of its thick struts and dense-core radial gradient, which concentrate material along the principal load paths, achieving 41.57 MPa compressive and 5.45 MPa tensile strength simultaneously. Lamella is the only topology to dominate both metrics under uniform porosity conditions. Schwarz-P (0.6 mm, $+0.3/0.0$) is optimal for residential and commercial panels because of its gradient-insensitive dominance across the phase diagrams: small fabrication deviations create negligible performance penalties. This is because its orthogonal channel architecture distributes loads isotropically in all three directions. Gyroid (0.6 mm, $-0.3/-0.3$) is best suited for seismic-prone regions because its continuous geometry enables distributed elastic deformation that absorbs seismic energy rather than transmitting it. It produces the lowest response amplification (0.03) and the lowest peak stress (0.1 MPa) of any topology tested. Diamond (0.6 mm, $-0.3/-0.3$) is best suited for non-structural applications that require extreme weight reduction. This equation was able to maintain connectivity at very thin wall thicknesses (60–75% void fractions) at which every other topology loses connectivity. Neovius (0.7–0.8 mm) is optimal for precision-equipment anchorages and drift-critical applications because of its high resistance to deformation—more so than any other structure. Neovius exhibits the highest experimental stiffness index (4.7 g/cm) and the lowest physical displacement (1.5 cm) during shaker-table testing. Lidinoid and Schwarz-G occupy intermediate performance territory, making them suitable for general construction applications such as partition walls or secondary structural elements, in which no single performance criterion

dominates, and broad mechanical adequacy across multiple load cases is the primary requirement.

Limitations and future directions

Four primary constraints bound the current findings. First, the Mazars continuum damage model captures brittle failure but neglects cyclic stiffness degradation under repeated loading. This highlights a need for further research on long-term seismic durability assessment with models accounting for fatigue and progressive microcrack accumulation across multiple earthquakes and loading cycles. Second, seismic simulations used synthetic ground motions at $\text{PGA} = 0.5g$; validation against recorded accelerograms from geographically diverse events (Northridge, 1994; Kobe 1995; Kahramanmaraş, 2023) across different soil conditions is required to confirm performance across the full range of seismic signatures. Third, the fixed-base boundary condition fails to account for soil–structure interaction. Future work should prioritize physical compressive and tensile testing of cement specimens fabricated via 3D concrete printing to validate the simulation framework, followed by cyclic loading tests to characterize fatigue behavior under repeated seismic excitation. Fourth, this research must continue to explore manufacturing pathways that enable democratic deployment of TPMS topologies.

While current literature shows that 3DCP would make these topologies directly printable and implementable today, other accessible alternatives are particularly well suited to post-disaster and resource-constrained contexts. First, printed formwork for cement casting: TPMS geometry is printed as a sacrificial polymer or clay mold into which conventional Portland cement is poured, requiring no specialized cement equipment on-site. This approach has already demonstrated a 65% reduction in slab mass at structural scale,⁴² and clay formwork has been shown to produce demoldable, building-scale concrete elements with complex geometries that extrusion-based printing cannot achieve.⁴³ Second, precast concrete panels manufactured off-site under controlled factory conditions can be rapidly transported and installed in disaster-affected areas. TPMS-optimized panels could be stockpiled in seismically active regions and deployed immediately following an earthquake, requiring only basic assembly labor on site. The inherently modular nature of homogeneous cellular theory would allow this approach without affecting performance.

Conclusions

This study demonstrates that topology-optimized TPMS geometries can simultaneously reduce cement consumption and enhance structural resilience without altering material chemistry, addressing cement's dual burden of high CO_2 emissions and brittle seismic failure. Across 8,750 simulated configurations spanning seven TPMS families, lamella emerged as the optimal topology, retaining 92.4% of solid concrete's compressive strength while using 32% less material, achieving 139.9% of the control's compressive

strength-to-weight ratio and an 180× improvement in tensile strength-to-weight ratio. All seven topologies achieved 85–98% reductions in seismic response amplification and 50–88% reductions in peak stress, shifting concrete's failure mode from brittle crushing toward distributed energy absorption—benefits that were independently corroborated by physical shaker-table testing. Sensitivity analysis confirmed that these rankings are geometry-driven and transfer across Portland cement formulations without recalibration, positioning this work as a mix-independent, widely implementable screening framework. The predictive ($R^2 = 0.987\text{--}0.999$) and generative machine learning frameworks further show that application-specific TPMS geometries, including entirely novel hybrid topologies outperforming the classical catalogue, can be identified without re-running full finite-element sweeps for every new design case. Because these benefits are realized at the material level—requiring no chemical reformulation, supplemental damping systems, or changes to existing supply chains and labor practices—TPMS-optimized cement geometries offer a scalable pathway to proportional CO₂ reduction and democratized seismic resilience for earthquake-prone, low-income communities worldwide.

Acknowledgments

During the preparation of this work, the authors used Cursor to streamline Python automation. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of this publication. We would like to express our deepest gratitude to our parents for supporting us through the long journey of developing this paper. Thank you for traveling with us across the United States to present at conferences, spending late nights practicing our speeches while balancing high school, and making this project financially possible. We also thank the teachers of HSMSE for supporting our journey in writing this paper, notably Dr. Gifty Asamani and Ms. Amneris Rasuk.

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